NEW TECHNIQUE USING POLES AND MODES DERIVATIVES FOR FREQUENCY AND GEOMETRY PARAMETERIZATION OF MICROWAVE STRUCTURES

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*Abstract*¾ **The present work describes a novel technique of parameterization for microwave circuit design and modeling in view of a fullwave 3D electromagnetic (EM) optimization. The proposed technique is based on the poles and modes computation using the finite element method and the use of the determined poles and modes for obtaining the transfer function characterizing the studied microwave structure frequency response over a large frequency band.**
The technique is then extended to geometry parameterization by computing the geometric derivatives of the determined poles and their corresponding modes. The computation of the derivatives allows establishing a very accurate parametric model describing the variation of the poles and the modes as a function of the circuit geometry deformation. Therefore, no more simulations or additional meshing are needed to evaluate the response of the circuit when its dimensions are changed.

I. Introduction

The fullwave 3D EM-field analysis and optimization of a microwave circuit require to perform several simulations using electromagnetic solvers based on the use of rigorous numerical methods like Finite Elements (FE) [1], or Finite Difference in Time Domain (FDTD). However, although the use of such methods is very accurate, each EM simulation is highly time consuming and optimizing using traditional techniques becomes a heavy task.

The major methods used for microwave optimization are based on establishing an equivalent electrical circuit that is used to evaluate the structure performances when their dimensions are tuned. However, in spite of the fact that this approach is very attractive due to the insignificant computer resources used for simulating the circuit, the major drawback is the lack of accurate electrical equivalent circuits models for complex structures.

In this work, our challenge is to establish an efficient method based on the rigorous EM simulation for frequency and geometry parameterization. For this purpose, we propose the use of poles expansion method which, not only reduces the CPU time consumed for studying a microwave circuit but also characterizes any circuit by its transfer function. Hence, the problem of establishing a global parametric model, which links the structure response to the frequency and the geometric parameters, becomes a problem of elaborating an accurate mapping between the position of poles and the circuit geometric parameters.

The parametric model is constructed by evaluating the Taylor polynomial expansion for each pole (see Fig. 1). For this purpose, our technique is based on the accurate evaluation of each pole derivatives.

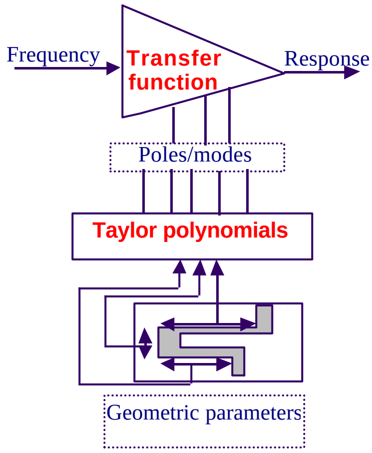

Fig.1. Parametric global model

II. Circuit transfer function using poles and modes

The projection of Maxwell equations using finite element method (FEM) leads to the general formulation [1]:

$$\left(R - k^2 M - jkF\right)e = jkh\sum_n J_{epn} \qquad (1)$$

Where R and M are the rigidity and the masse matrices respectively. The formulation (1) can be written in the global form of a linear system as: $A(k).e = B(k)$. The modes of the non-excited structure are obtained for the values of k which make the matrix $A(k)$ singular. In the case of a lossless structure, these values are given by:

$$R\,.\,V_i\ -\ k_i^2 M\,.\,V_i = 0 \qquad (2)$$

The k_i values and the V_i vectors are the system-generalized eigenvalues and eigenvectors, which correspond to the resonant frequencies and the cavity modes.

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Different well-known algorithms as the Arnoldi-Lanczos one [2] can perform solving the system given in (2). Applying the Kurokawa principle [3], the E or the H-fields propagating inside the cavity can be expressed as a decomposition on the cavity modes. Hence, we can write the E-field solution, for each excited port noted by n as:

$$e_n = \sum_i a_{in}(k).V_i \quad (3)$$

Using (1), (2) and (3) the coefficients $a_{ni}(k)$ can be determined and the E-field can be given by the following expression:

$$e_n = jh\,k \sum_i \frac{C_{ni}}{k_i^2 - k^2} V_i \quad (4)$$

Where:

- $C_{ni} = \langle J_{epn} / V_i \rangle$ (scalar product)
- J_{epi} : n^{th} Port excitation
- V_i : cavity mode vector

Using (4), the impedance matrix [Z] can be calculated, since each impedance term, associated to the ports noted n and m, is given by the relation ($Z_{nm} = {}^tJ_{epn}\, e_m$) [1]. Therefore, the pole expansion of the impedance matrix can be written in the form:

$$Z_{nm} = jh\,k \sum_i \frac{C_{ni} C_{mi}}{k_i^2 - k^2} \quad (5)$$

The formulation given in (5) is the global transfer function characterizing the studied microwave structure expressed in terms of a pole expansion. However, some poles appear at the zero frequency and their computation is highly time consuming. For that reason, the zero poles are separated from others and the expression (5) becomes:

$$Z_{nm} = \frac{A_{nm}}{jkh} + jkhB_{nm} + jhk^3 \sum_{i=1}^{Q} \frac{C_{ni}C_{mi}}{k_i^2(k_i^2 - k^2)} \quad (6)$$

$$\text{Where:} \quad \begin{cases} A_{nm} = \sum_{i=1}^{Q} C_{ni}C_{mi} \\ B_{nm} = \sum_{i=1}^{Q} \frac{C_{ni}C_{mi}}{k_i^2} \end{cases} \quad (7)$$

The quantities A_{mn} and B_{mn} convergence slowly [6]. They depend on the type of excitation so we determine them using asymptotic approximations.

For example, for rectangular waveguides junctions having n excited ports, they are determined using the following relation :

$$Z_{nm} \to d_{nm} Z_p, \quad k \to 0 \quad (8)$$

$$\text{Where :} \quad \left. \begin{array}{ll} Z_p \approx \frac{jkh}{k_a} & \text{for TE mode} \\ Z_p \approx \frac{k_a h}{jk} + \frac{jkh}{2k_a} & \text{for TM mode} \\ k_a = p/a & \end{array} \right\} \quad (9)$$

This relation is valid assuming that the reference planes taken on the problem ports are sufficiently far from the studied junction (not less than half wavelength).

Comparing (6) and (8) we obtain:

$$Z_{nm} = d_{nm} h \left(\frac{c_n k_n}{jk} + \frac{jk}{(1 + c_n) k n} \right) + jhk^3 \sum_i \frac{c_{ni} c_{mi}}{k_i^2 (k_i^2 - k^2)} \quad (10)$$

- χ_n is 0 or 1 depending on TE or TM excitation mode respectively.
- δ_{nm} : Kroneker symbol.
- k_n wavenumber corresponding to n^{th} pole.
- $C_{ni} = \langle J_{epi} / V_n \rangle$.

The last formula demonstrates that the computation of the finite number of poles and their corresponding modes suffice to characterize the harmonic behavior of any circuit. Practically, to have a good accuracy, the number of required poles "N" depends on the frequency studied range [f_{min} - f_{max}]. A good approximation can be achieved when the largest calculated pole position is situated greater than twice the upper frequency band ($f_N > 2f_{max}$) [4]. Furthermore, the last formulation demonstrates that the computation of poles and their corresponding modes allows to obtain analytically the broad band harmonic behavior for the studied microwave circuit.

III. Poles and modes derivatives

Many methods have been used for constructing a mapping that takes into account the geometric variation of a circuit [5]. In this paper, we propose the use of Taylor expansion to build a relationship between each mode and pole on one side and the geometric parameters on the other side. Therefore, each pole or mode is parameterized using the Taylor polynomials :

$$x(p + \Delta p) = x(p) + \sum_{n>0} \Delta p^n \frac{x^{(n)}(p)}{n!} \quad (11)$$

Where "p" is the geometric parameter vector and "x" is the considered pole or mode.

To build the required Taylor polynomial, we have first to compute the derivatives denoted $x^{(n)}(p)$. For this purpose, let's consider the equations verified by each pole and its corresponding mode:

$$R(p)V(p) - l(p)M(p)V(p) = 0 \quad (12a)$$

$$^tV(p)M(p)V(p) = 1 \quad (12b)$$

Where, λ is a mathematical notation for the pole (corresponding to k^2). Deriving equation (12) to the order "m" and using the Leibnitz formula we obtain the following system:

$$(R - lM)v^m - l^m Mv = -\sum_{i=1}^{m} (R^i - lM^i) v^{m-i} + \sum_{i=1}^{m-1} l^i (Mv)^{m-i} \quad (13.a)$$

$$-{}^t vM.v^{(m)} = \frac{1}{2} \left[\sum_{i=1}^{m} {}^t vM^{(i)} v^{(m-i)} + \sum_{i=1}^{m-1} {}^t v^{(i)} (Mv)^{(m-i)} \right] \quad (13.b)$$

(i.e. For all the derivatives we note $x^{(k)}(p) = \frac{1}{k!}\frac{\partial^k x}{\partial p^k}$).

The modes and poles derivatives are then computed using the iterative system given in (13a) and (13b). The derivatives of R and M matrices describing the structure mesh are obtained using an automatic differentiation technique [6].

IV. Test cases

IV.1. Band pass filter

This test is performed for studying the influence of two predefined geometric parameters and the frequency on the scattering parameters of a band pass waveguide filter. The dimensions of the circuit are shown in Figure 2.

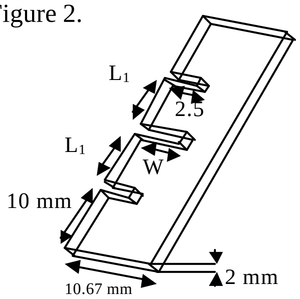

Fig. 2. Three irises Band Pass filter

The chosen geometrical parameters are the iris length “w” and the cavity length “L_1”. The parameters variation ranges are given in Table 1.

Table 1. Band pass filter parameters ranges

	Min	Initial	Max
w [mm]	2	3.13	4
L_1 [mm]	8	10	12
Frequency [GHz]	17	-	22

Twenty modes are needed to calculate the scattering parameters S_{11} and S_{12}. A six order geometric derivation is used to construct the multi-parametric Taylor polynomial model describing the variation of the modes as a function of the geometric parameters. Once the Taylor expansion is built for each mode and pole, only few seconds on a workstation are needed to determine the pole positions and their associated modes for any value of the geometric parameters (“L1” and “w”) inside the predefined range. Figure 3 shows the ten first poles positions when “L_1” is varied. The results are compared to those obtained using direct computation of the poles using an adequate meshing of the structure corresponding to each variation of "L_1".

From these results, it can be seen that one can easily design a filter and vary its pass frequency band, by varying the positions of the interesting modes.

Furthermore, using the pole expansion method applied to the determined poles and modes enables to determine rapidly and accurately the harmonic response of the circuit for any value for the geometric parameters inside their respective variation ranges. Figure 4 and Figure 5 show the variation of the return loss S_{11} as a function of “w” and “L_1” respectively using our proposed technique. A very good agreement is noticed between these results and those obtained using a direct computation for each value of the geometric parameters.

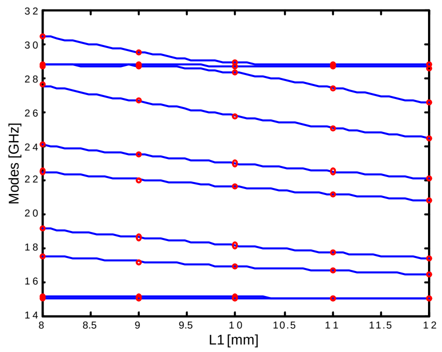

Fig. 3. Modes variation as a function of “L_1“ for the waveguide band pass filter. Comparison between direct computation (doted circles) and results using our technique (continued curves

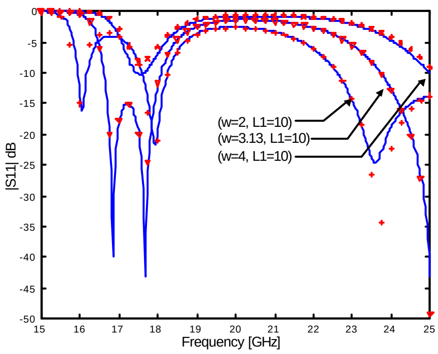

Fig. 4. Variation of S_{11} as a function of frequency using “W” as a parameter. (doted points) direct computation (continued curves) our technique

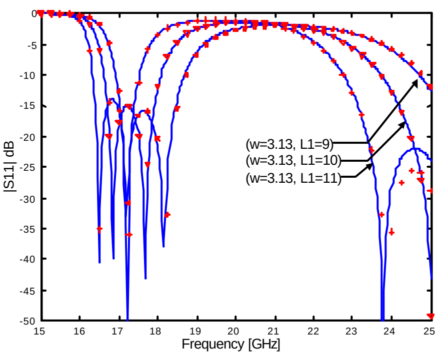

Fig. 5. Variation of S_{11} as a function of frequency using “L_1” as a parameter. (doted points) direct computation (continued curves) our technique.

IV.2. Cavity resonator

This test is performed to compare our theoretical results to those obtained from measurements. The fabricated structure is a cylindrical cavity having a resonator inside it. Figure 17 gives the studied structure having "r" and "d" as geometrical parameters. The parameters variation ranges are given in Table 2.

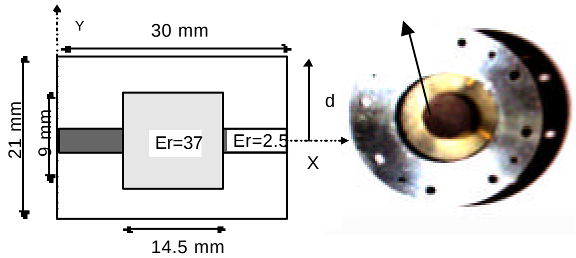

Fig.6. Dielectric cavity resonator

Table 2. Cavity resonator parameters ranges

	Min	Initial	Max
r [mm]	13	15	17
d [mm]	7	10.5	12
Frequency [GHz]	2		5

The considered modes are denoted TM_{01x}, TE_{01X}, EH_I and EH_{II} (see Figure 7) according to their analogue form to modes obtained for an air-cylindrical cavity.

Figure 8 gives the modes variation obtained using a Taylor 6th order expansion compared to the measurements results when varying the parameter "d". This figure shows a very good agreement between results using our technique and the experimental ones.

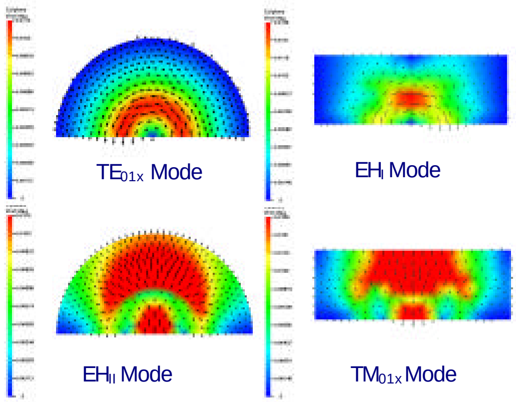

Fig. 7. Determined modes for the dielectric cavity resonator

V. Conclusion

A new method has been proposed for frequency and geometry parameterization that can be efficiently used for circuit design and optimization in the frequency domain using the finite element method. It combines a pole expansion of the EM fields and the derivatives computation to build an accurate multi-parametric model for both frequency and geometry parameters.

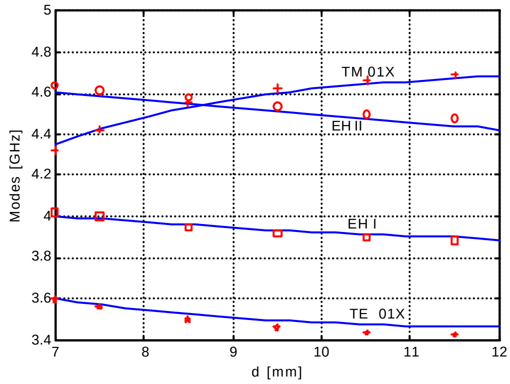

Fig. 8. Mode variations as a function of the parameter "d". Comparison between Taylor 6 th order expansion (filled line) and measurements (circles)

The characterization of the frequency response by a transfer function defined by its poles and modes decreases the CPU time required for simulating a microwave circuit. This is because only a few number of modes is needed to accurately fit the response. Furthermore, the Taylor expansion applied on the calculated modes allows to judge how one can modify the circuit dimensions according to a required response. Once the modes and poles mappings are established, one can rapidly study the influence of any circuit dimensions variations on the harmonic response without any need of repetitive mesh creation or EM rigorous simulations.

VI. References

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